

Circle of radius R.

String tied from top "P" to location "Q".

Bead slides down string without friction.

- a) How long does bead take to slide to Q?

We need to know distance travelled + acceleration.

Acceleration is $a = g \cos \theta$

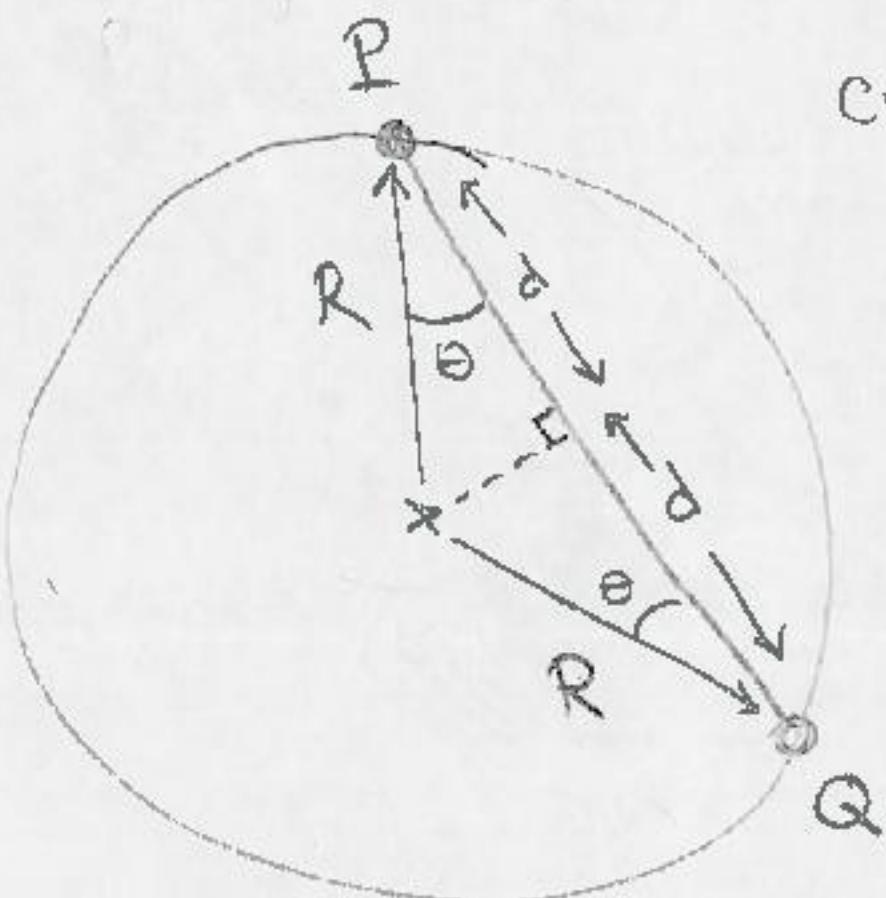
Create identical right triangles, as shown. The side of triangle

$$d = R \cos \theta$$

so total distance travelled is

$$\text{dist} = 2d$$

$$= 2R \cos \theta$$



Now use kinematics to compute time it takes to slide from P to Q:

$$\text{dist} = v_0 t + \frac{1}{2} a t^2$$

$$2R \cos\theta = \frac{1}{2} (g \cos\theta) t^2$$

$$t^2 = \frac{4R}{g}$$

$$\rightarrow t = 2\sqrt{\frac{R}{g}}$$

Note time does not depend on angle θ

- b) Speed at point Q can be found via kinematics, or via conservation of energy.

$$\begin{aligned}\text{Kinematics: } v_f^2 &= v_0^2 + 2a(\text{dist}) \\ &= 2(g \cos\theta)(2R \cos\theta) \\ &= 4gR \cos^2\theta\end{aligned}$$

$$\rightarrow v_f = 2 \cos\theta \sqrt{gR}$$

$$\begin{aligned}\text{energy } \frac{1}{2}mv_f^2 &= mgh = mg(2R \cos\theta) \cos\theta \\ &= 2mgR \cos^2\theta\end{aligned}$$

$$\rightarrow v_f^2 = 4gR \cos^2\theta$$

$$v_f = 2 \cos\theta \sqrt{gR}$$

