Alice sets up oscillators and microphonos in her audio lab. She records at first only with microphone A.

oscillator #1 alone: $y_1(t) = 12.8\,\mu Pa \cos\left(1495.4\,\frac{rad}{s}\cdot t\right)$

a) wavelength is given by

$$V_s = \lambda \cdot f$$

where $V_s = 343\,\frac{m}{s}$. So the wavelength must be

$$\lambda_1 = \frac{V_s}{f} = \frac{V_s}{\omega/2\pi} = \frac{343\ m/s}{\left(1495.4\ rad/s \,/\, 2\pi\ rad\right)}$$

$$\lambda_1 = 1.44\ m$$

The wave created by oscillator #2 alone is

$$y_2(t) = 12.8\,\mu Pa \cos\left(1534.4\,\frac{rad}{s}\cdot t\right)$$

Note that the frequencies of the oscillators are not exactly the same – but they are nearly the same. That means we can express the sum of the waves as

$$y_3(t) = y_1(t) + y_2(t)$$

$$= 2A\cos\left(\frac{\omega_1+\omega_2}{2}t\right)\cos\left(\frac{\omega_1-\omega_2}{2}t\right)$$

In this case,

$$y_3(t) = \underbrace{25.6\ \text{Pa}}\cos\left(1514.9\ \frac{\text{rad}}{\text{s}}\cdot t\right)\cos\left(19.5\ \frac{\text{rad}}{\text{s}}\cdot t\right)$$

So

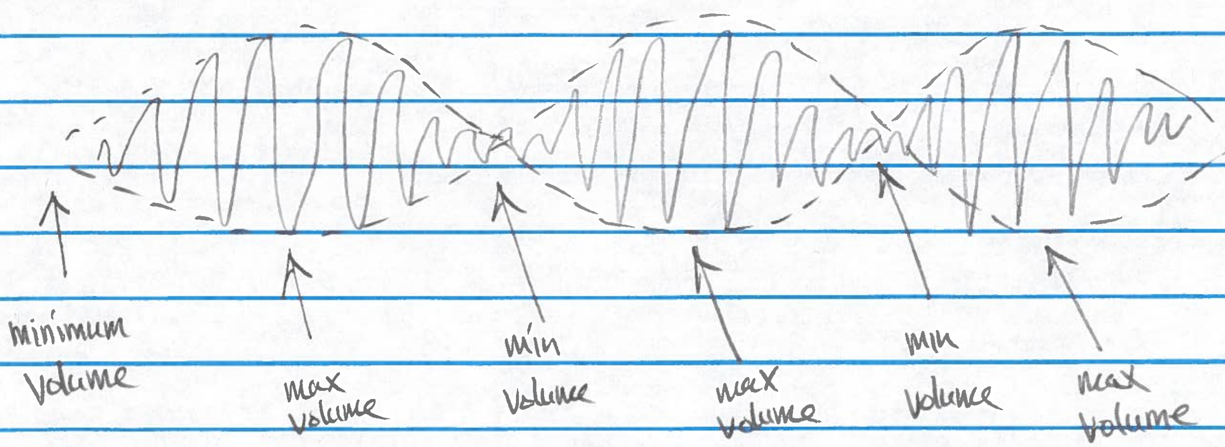
- max amplitude = 25.6 Pa
- beat frequency = $f_{beat} = |f_1 - f_2|$

$$\text{but } f_1 = \frac{\omega_1}{2\pi} = 238\ \text{Hz}$$

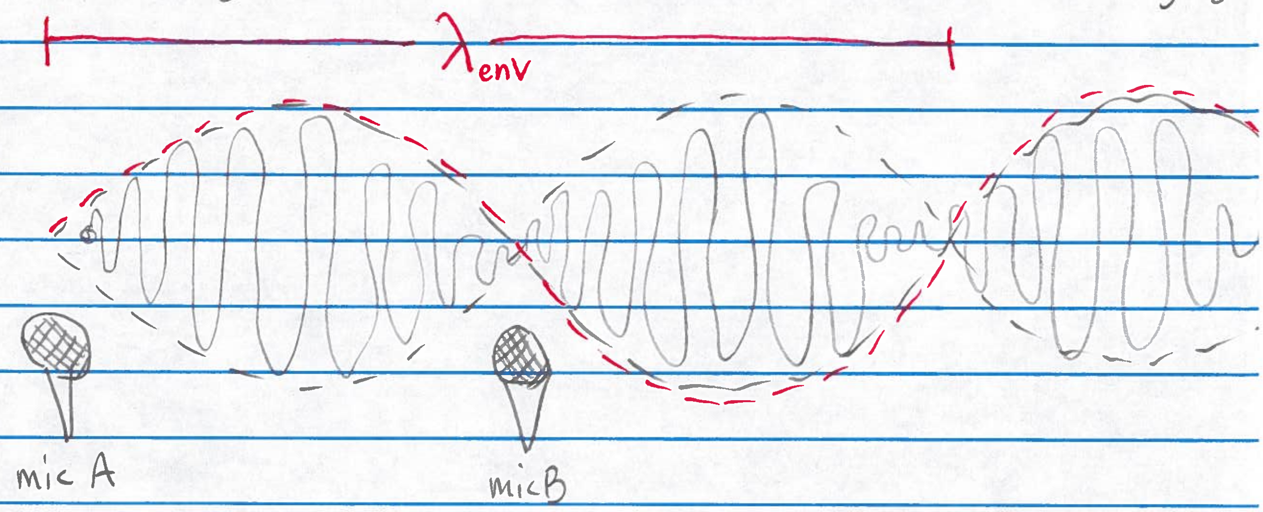
$$f_2 = \frac{\omega_2}{2\pi} = 244.2\ \text{Hz}$$

$$\Rightarrow f_{beat} = 6.2\ \text{Hz}$$

The shape of the combined wave looks like this



How far apart should Alice place the two microphones so that they record a minimum sound level simultaneously?



The envelope wave, shown in red, has 2 minima per cycle — two places where the sound volume is smallest. The envelope's wave function is

$$y_{env}(t) = \cos\left(19.5 \tfrac{rad}{s} \cdot t\right)$$

So

$$\omega_{env} = 19.5 \tfrac{rad}{s}$$

The speed of sound waves is

$$V_s = 343 \tfrac{m}{s} = \frac{\omega_{env}}{K_{env}}$$

$$\rightarrow K_{env} = \frac{\omega_{env}}{V_s} = \frac{19.5 \tfrac{rad}{s}}{343 \tfrac{m}{s}} = 0.0569 \tfrac{rad}{m}$$

So the wavelength of the envelope is

$$\lambda_{env} = \frac{2\pi \text{ rad}}{0.0569 \frac{\text{rad}}{\text{m}}}$$

$$= 110 \text{ m}$$

As the diagram shows, the volume of the combined wave has minimum values every half-wavelength, so

$$D = \frac{1}{2}\lambda_{env} = 55 \text{ m}$$