



Joe stretches a string of mass $m = 2.5 \text{ kg}$ and length $L = 100 \text{ m}$ between two posts.

$$\mu = \frac{m}{L} = \frac{2.5 \text{ kg}}{100 \text{ m}} = 0.025 \text{ kg/m}$$

He whacks one end, creating a wave with amplitude $A = 0.018 \text{ m}$. The crests are $\lambda = 1.1 \text{ m}$ apart, and the speed of the wave along the string is $V_{\text{wave}} = 18.6 \text{ m/s}$.

Use V_{wave} and k to find angular speed ω .

$$\omega = V_{\text{wave}} \cdot k = \left(18.6 \frac{\text{m}}{\text{s}}\right) \left(\frac{2\pi \text{ rad}}{1.1 \text{ m}}\right)$$

$$\omega = 106 \frac{\text{rad}}{\text{s}}$$

Now, if

$$y(x,t) = A \cos(kx - \omega t)$$

$$v_y(x,t) = \frac{dy}{dt} = \omega A \sin(kx - \omega t)$$

So the maximum vertical speed is

$$\begin{aligned} \max v_y &= \omega A = \left(106 \frac{\text{rad}}{\text{s}}\right) (0.018 \text{ m}) \\ &= 1.91 \frac{\text{m}}{\text{s}} \end{aligned}$$

Likewise, we can find the vertical acceleration of the string by taking the second time derivative:

$$a_y = \frac{d^2 y}{dt^2} = -\omega^2 A \cos(kx - \omega t)$$

The maximum value of a_y is

$$\begin{aligned} \max a_y &= \omega^2 A = \left(106 \frac{\text{rad}}{\text{s}}\right)^2 (0.018 \text{ m}) \\ &= 203 \frac{\text{m}}{\text{s}^2} \end{aligned}$$

The power carried by this wave is

$$\begin{aligned} \text{Power} &= \frac{1}{2} (\mu v_{\text{wave}}) \omega^2 y_{\text{max}}^2 \quad \left\{ \begin{array}{l} \text{same as} \\ \text{amplitude } A \end{array} \right. \\ &= \frac{1}{2} \left(0.025 \frac{\text{kg}}{\text{m}}\right) \left(18.6 \frac{\text{m}}{\text{s}}\right) \left(106 \frac{\text{rad}}{\text{s}}\right)^2 (0.018 \text{ m})^2 \\ &= 0.85 \text{ Watts} \end{aligned}$$