

Long thin rod of uniform density has

$$\text{mass } M = 2.35 \text{ kg}$$

$$\text{length } L = 1.6 \text{ m}$$

The rod can pivot around point $P = (0,0)$.

Moment of inertia of rod around pivot is

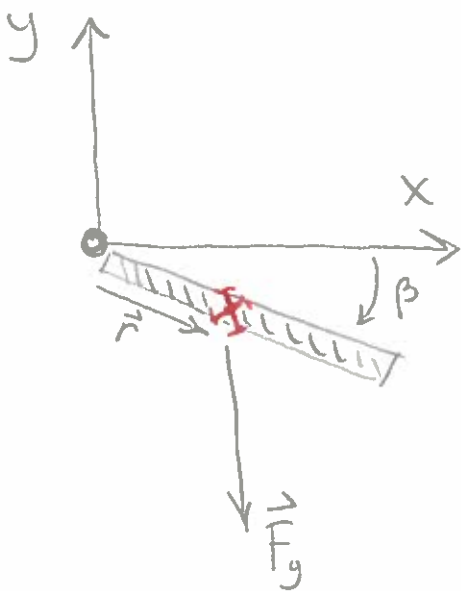
$$I = \frac{1}{3} ML^2 = 2.005 \text{ kg}\cdot\text{m}^2$$

Center of mass is at the geometric center of the rod, marked "X" in picture above

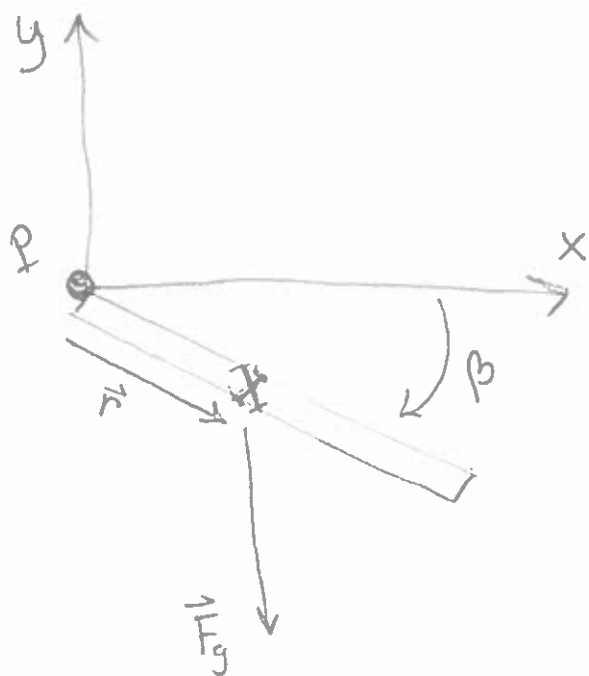
$$\text{CoM} = \frac{L}{2} \hat{i} + 0 \hat{j} = 0.8 \text{ m} \hat{i} + 0 \text{ m} \hat{j}$$

Gravity pulls on the rod at its center of mass, so the torque on rod around point P is

$$\begin{aligned} \tau &= \vec{r} \times \vec{F}_g = (0.8 \text{ m} \hat{i} + 0 \text{ m} \hat{j}) \times (-23.03 \text{ N} \hat{j}) \\ &= -18.4 \text{ N}\cdot\text{m} \hat{k} \quad \text{into page} \end{aligned}$$



The rod is released, and begins to swing down. After a short time, it has rotated by angle $\beta = 20^\circ$.



The center of mass is now at

$$x = r \cos \beta = \frac{L}{2} \cos \beta$$

$$= 0.752 \text{ m}$$

$$y = -r \sin \beta = -\frac{L}{2} \sin \beta$$

$$= -0.274 \text{ m}$$

The change in GPE from initial state is

$$\Delta \text{GPE} = \text{GPE}_f - \text{GPE}_i$$

$$= mgy_f - mgy_i$$

$$= (2.35 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})(-0.274 \text{ m}) - 0$$

$$= -6.30 \text{ J}$$

The kinetic energy of the rod is equal in size to this

$$\text{KE}_i + \text{GPE}_i = \text{KE}_f + \text{GPE}_f$$

$$0 + 0 = \frac{1}{2} I \omega_f^2 + (-6.30 \text{ J})$$

$$\Rightarrow \frac{1}{2} I \omega_f^2 = 6.30 \text{ J}$$

So the angular velocity is

$$\omega_f = \sqrt{\frac{6.30 \text{ J}}{\frac{1}{2} (2.005 \text{ kg} \cdot \text{m}^2)}} = 2.51 \frac{\text{rad}}{\text{s}}$$