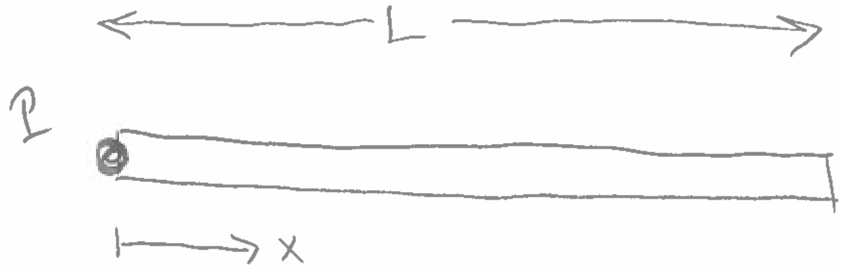
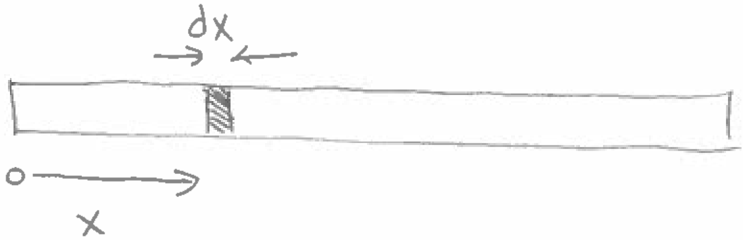


A wooden rod sits on an ice rink; it is pinned to ice at the left end, at point P. $L = 1.8\,m$.

The rod has a density which varies with position

$$\lambda(x) = 0.29\,\frac{kg}{m} + 0.073\left(\frac{x}{L}\right)^2 \frac{kg}{m}$$



Consider one little piece of the rod, a distance x from the left end, and of width dx.

$$\text{mass of piece} \quad dm = \lambda(x)\,dx$$

$$= 0.29\,\frac{kg}{m}\,dx + \frac{0.073}{L^2}\,\frac{kg}{m}\,x^2\,dx$$

So we can find mass of entire rod by integrating

$$m = \int dm = \int_{x=0}^{x=L} 0.29\,\frac{kg}{m}\,dx + \int_{x=0}^{L} \frac{0.073}{L^2}\,\frac{kg}{m}\cdot x^2\,dx$$

$$= 0.29\,\frac{kg}{m}\left(x\Big|_{x=0}^{x=1.8m}\right. + \frac{0.073}{(1.8\,m)^2}\,\frac{kg}{m}\left(\frac{1}{3}x^3\Big|_{x=0}^{x=1.8m}\right.$$

$$= 0.522\,kg + 0.0225\,\frac{kg}{m^3}\cdot\left(1.944\,m^3\right)$$

$$= 0.522\,kg + 0.0438\,kg$$

$$\boxed{m = 0.566\,kg}$$

The moment of inertia of the rod around point P. is given by a similar technique. The moment of inertia for one small piece is

$$dI = dm \cdot x^2$$

$$= \left(0.29\,\tfrac{kg}{m}\,dx + \frac{0.073}{L^2}\,\tfrac{kg}{m}\cdot x^2\,dx\right)x^2$$

$$= 0.29\,\tfrac{kg}{m}\,x^2\,dx + \frac{0.073}{L^2}\,\tfrac{kg}{m}\cdot x^4\,dx$$

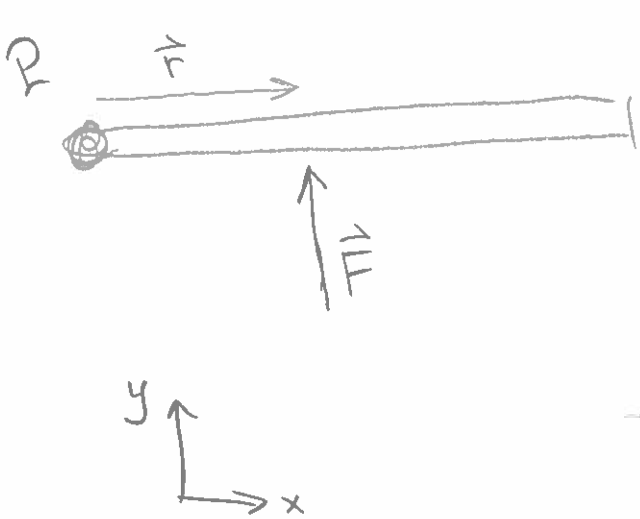
So the total moment of inertia is

$$I = \int dI = \int_{x=0}^{x=1.8\,m} 0.29\,\tfrac{kg}{m}\,x^2\,dx + \int_{x=0}^{x=1.8\,m} \frac{0.073}{(1.8\,m)^2}\,\tfrac{kg}{m}\,x^4\,dx$$

$$= 0.29\,\tfrac{kg}{m}\left(\tfrac{1}{3}x^3\Big|_0^{1.8m}\right) + 0.02251\,\tfrac{kg}{m^3}\left(\tfrac{1}{5}x^5\Big|_0^{1.8m}\right)$$

$$= 0.5638\ kg\cdot m^2 + 0.0851\ kg\cdot m^2$$

$$\boxed{I = 0.6489\ kg\cdot m^2}$$



Jack kicks the rod halfway from pin to right end with a force $\vec{F} = 59\ N$ as shown.

$$\vec{r} = 0.9\ m\ \hat{i}$$

$$\vec{F} = 59\ N\ \hat{j}$$

The torque due to this force, around the point P, is

$$\vec{\tau} = |r||F| \sin 90° \quad \text{out of page}$$

$$= 53.1 \text{ N}\cdot\text{m} \quad \text{out of page}$$

As a result, the rod starts to rotate around point P.
The angular acceleration is

$$\vec{\alpha} = \frac{\vec{\tau}}{I} = \frac{53.1 \text{ N}\cdot\text{m} \text{ out of page}}{0.6489 \text{ kg}\cdot\text{m}^2}$$

$$= 81.8 \frac{\text{rad}}{\text{s}^2} \quad \text{out of page}$$