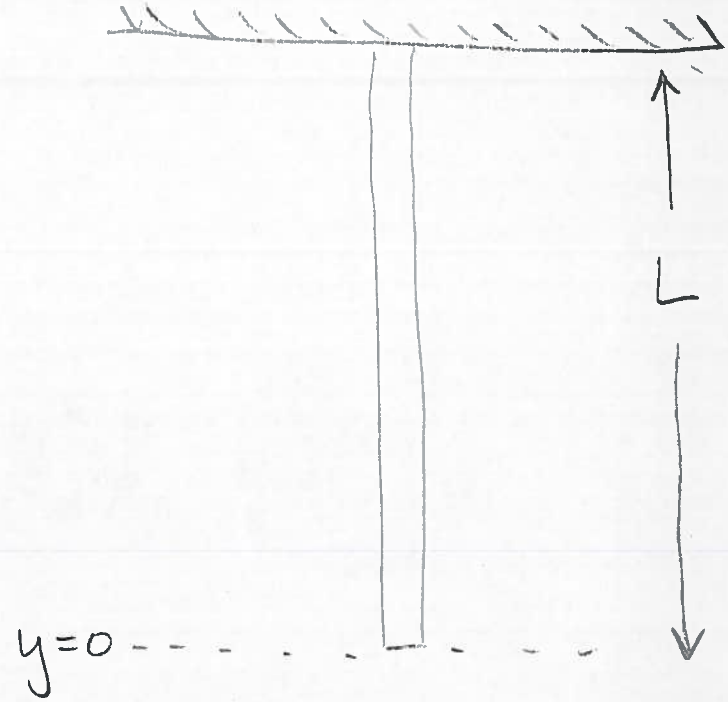
Rope of length $L = 100\ m$ and mass $m = 5\ kg$ hangs from ceiling.



Linear mass density

$$\mu = \frac{m}{L} = 0.05\ \frac{kg}{m}$$

a) mass of rope hanging below height y is

$$m(y) = \mu y$$

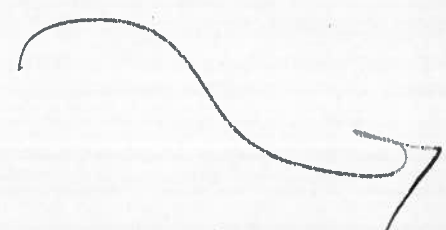
b) tension of rope at height y is due to force of gravity on the portion of rope below point y. So

$$\text{tension } T(y) = m(y) * g = \mu y g$$

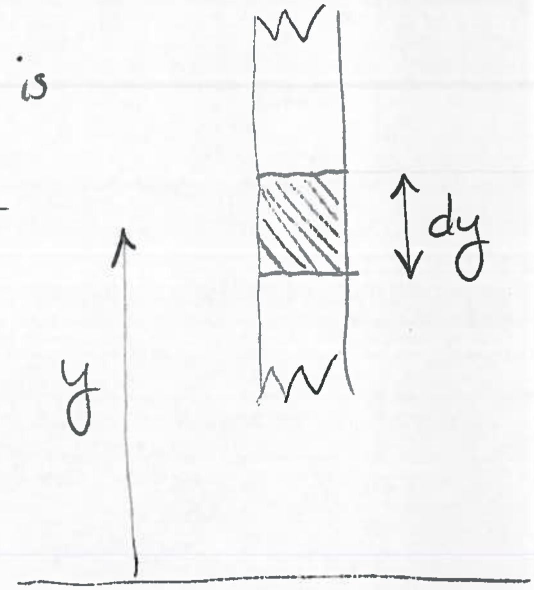
c) speed of wave travelling along rope at height y is

$$v(y) = \sqrt{\frac{T(y)}{\mu}} = \sqrt{\frac{\mu y g}{\mu}} = \sqrt{yg}$$

Suppose we hit rope in the middle, at $y = 50\ m$ from top or bottom. Waves will travel up and down.



Time for wave to travel <u>up</u> rope is given by this idea: look at one little piece of rope, at height y, of size dy.



speed of wave $= \sqrt{gy}$

distance wave must travel through piece $= dy$

time it takes to go through piece is

$$dt = \frac{\text{distance}}{\text{speed}} = \frac{dy}{\sqrt{gy}}$$

So time to go up to top is

$$t_{up} = \int_{y=50m}^{y=100m} dt = \int_{y=50m}^{100m} \frac{dy}{\sqrt{gy}} = \frac{1}{\sqrt{g}} \int_{50m}^{100m} y^{-1/2}\, dy$$

$$= \frac{1}{\sqrt{g}} \left[2y^{1/2} \right|_{50m}^{100m} = 1.87 \text{ sec}$$

the winner!

Likewise

$$t_{down} = \frac{1}{\sqrt{g}} \left[-2y^{1/2} \right|_{50m}^{0m} = 4.52 \text{ sec}$$