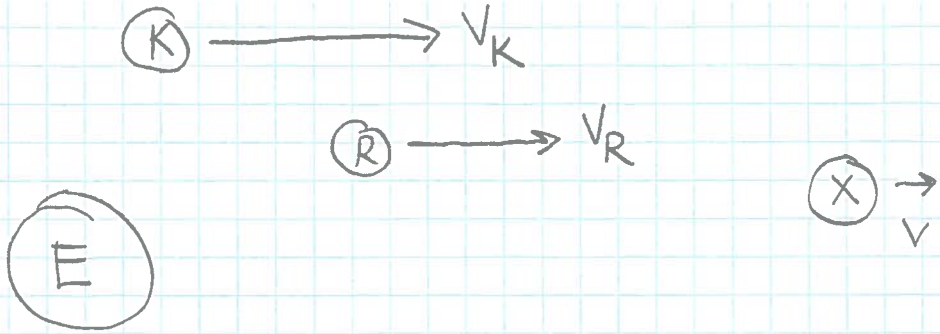



Earth sees object of mass $M = 7.35 \times 10^{22}$ kg, moving at speed $V = 10^4$ m/s.

$$V << c \rightarrow \gamma \approx 1$$

$$E = \gamma mc^2 \approx mc^2 = 6.61 \times 10^{39} \text{ J}$$

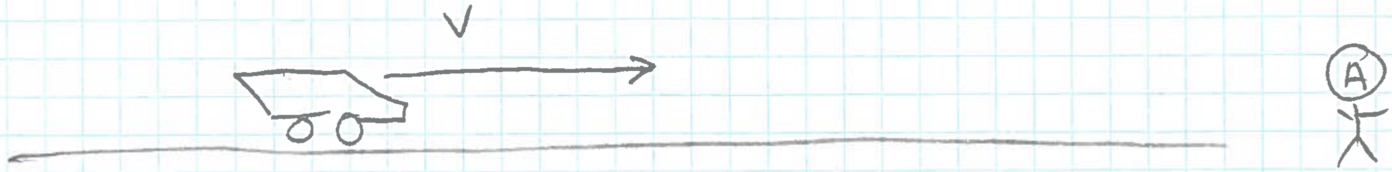
$$p = \gamma mv = 7.35 \times 10^{26} \text{ kg·m/s}$$

$$ME = \sqrt{E^2 - p^2c^2} \approx E = 6.61 \times 10^{39} \text{ J}$$

Romulans are travelling at $V_R >> V$, Klingons at $V_K >> V$. Each will see the object moving at approximately $V_R$ and $V_K$, respectively. Each should measure a different E and p, but get the same momenergy as Earth observers.

| | Romulan | Klingon |
|---|---|---|
| E | $7.63 \times 10^{39}$ J | $1.52 \times 10^{40}$ J |
| p | $1.27 \times 10^{31}$ kg·m/s | $3.25 \times 10^{31}$ kg·m/s |
| $\sqrt{E^2 - p^2c^2}$ | $6.61 \times 10^{39}$ J | $1.17 \times 10^{40}$ J |

Klingon reported ME is not same as that of the Earth ... so Klingons are lying!





A car travels at speed v to the East along road. It emits radio waves of frequency $f = 2.4$ GHz.

Suppose $v = 0.7c$. What are frequencies measured by police officers A (to East) and B (to South)?

$$A: \quad f' = f\frac{\sqrt{1+v/c}}{\sqrt{1-v/c}} = 2.38\, f = 5.7 \text{ GHz}$$

$$B: \quad f' = f\frac{1}{\gamma} = f\sqrt{1-\frac{v^2}{c^2}} = 0.71\, f = 1.7 \text{ GHz}$$

If car travels at a small relativistic speed, $v \ll c$, then

$$A: \quad f' = f\frac{\sqrt{1+v/c}}{\sqrt{1-v/c}} = f\left(1+\frac{v}{c}\right)^{1/2}\left(1-\frac{v}{c}\right)^{-1/2}$$

$$\cong f\left(1+\frac{1}{2}\frac{v}{c}\right)\left(1+\frac{1}{2}\frac{v}{c}\right) \approx f\left(1+\frac{v}{c}\right)$$

$$B: \quad f' = f\frac{1}{\gamma} = f\sqrt{1-\frac{v^2}{c^2}} = f\left(1-\frac{v^2}{c^2}\right)^{1/2} \approx f\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)$$

So the <u>difference</u> in frequency

$$|\Delta f| = |f' - f| = \begin{cases} \approx f\dfrac{v}{c} & \text{for A} \quad \underline{\text{bigger}} \\ \approx f\left(\dfrac{1}{2}\dfrac{v^2}{c^2}\right) & \text{for B} \end{cases}$$

Joe drives at $v = 50$ mph for 60 years. How much "extra life" does he gain?

$$V = 50 \text{ mph} * \frac{1609 \text{ m}}{\text{mile}} * \frac{1 \text{ hour}}{3600 \text{ s}} = 22.3 \frac{\text{m}}{\text{s}}$$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \cong 1 - \left(-\frac{1}{2}\right)\frac{v^2}{c^2} \cong 1 + \frac{1}{2}\frac{v^2}{c^2}$$

$$= 1 + 2.8 \times 10^{-15}$$

So

$$\Delta t_{Joe} = \frac{1}{\gamma} \Delta t_{normal}$$

$$= \Delta t_{normal} \left(1 + 2.8 \times 10^{-15}\right)^{-1}$$

$$\cong \Delta t_{normal} \left(1 - 2.8 \times 10^{-15}\right)$$

Now, $\Delta t_{Joe} = 60 \text{ years} = 1.89 \times 10^9 \text{ s}$

$$\Rightarrow \Delta t_{Joe} \cong \Delta t_{normal} - 2.8 \times 10^{-15} \Delta t_{normal}$$

$$\left(\Delta t_{normal} - \Delta t_{Joe}\right) = \text{"extra life"} \cong 2.8 \times 10^{-15} \left(\Delta t_{normal}\right)$$

$$= 2.8 \times 10^{-15} \left(1.89 \times 10^9 \text{ s}\right)$$

$$= \underline{5 \text{ microseconds}}$$

a) Biker travels 5 m of space in 11 m of time

$$V = \frac{5\,m}{11\,m} = 0.455\,c \qquad \gamma = 1.123$$

b) $t_A - t_B = 0$ according to Bob

c) Bob measures $A = (-11, -9)$ $B = (2, -9)$

To compute Biker's location of each event, use Lorentz transformation - note one small complication

$$A: \quad x' = \gamma(x - vt) = 1.123\left(-11 - (0.455)(-9)\right)$$
$$= -7.76$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = 1.123\left(-9 - \frac{(0.455)(-11)}{1^2}\right)$$
$$= -4.49$$

<u>But</u> Biker's clock is offset from Jane + Bob's clock by +5.

$$\rightarrow t' = -4.49 + 5 = 0.51$$

We can likewise compute location of event B acc. to Biker.

$$B: \quad x' = \gamma(x - vt) = 1.123\left(2 - (0.455)(-9)\right)$$
$$= 6.83$$
$$t' = 5 + \gamma\left(t - \frac{vx}{c^2}\right) = 5 + 1.123\left(-9 - \frac{(0.455)(2)}{1^2}\right)$$
$$= -6.12$$

Thus we find the interval in time between A and B acc. to Biker is

$$\Delta t' = t'_A - t'_B = 0.51 - (-6.12) = \boxed{6.63\text{ sec}}$$

As a check, compute the STI for each observer.

$$\text{Bob: } STI = \sqrt{(t_A - t_B)^2 - (x_A - x_B)^2}$$

$$= \sqrt{(0)^2 - (-13)^2} = \underline{\underline{13\,i\ m}}$$

$$\text{Biker: } STI = \sqrt{(t'_A - t'_B)^2 - (x'_A - x'_B)^2}$$

$$= \sqrt{(6.63)^2 - (-7.76 - 6.83)^2} = \underline{\underline{13\,i\ m}}$$

Yes, same STI. Note that it is imaginary, so no message can pass between events Ⓐ and Ⓑ.

d) time interval acc. to Jane: $t_C - t_B = 9\ m$

e) Once again, use Lorentz trans for Biker:

$$C: \quad x' = \gamma(x - vt) = 1.123\,(2 - (0.455)(0))$$
$$= 2.25\ m$$

$$t' = 5 + \gamma\left(t - \frac{vx}{c^2}\right) = 5 + 1.123\left(0 - \frac{(0.455)(2)}{1^2}\right)$$
$$= 3.98\ m$$

$$\rightarrow \text{Biker } (t'_C - t'_B) = 3.98 - (-6.12) = \boxed{10.1\ m}$$

f) Biker flashes light at point Ⓕ, beam travels to Bob and reaches him at

$$F: (-11, 2)$$

We need to use geometry and algebra to determine the location of event Ⓕ. →

Equation of flash of light: $t = -x - 9$

Equation of Biker: $x = \frac{5}{11} t + 2$

Intersection when these are equal

$$x = \frac{5}{11}(-x-9) + 2$$

$$x = -\frac{5}{11}x - \frac{45}{11} + 2$$

$$\frac{16}{11}x = -\frac{45}{11} + \frac{22}{11}$$

$$16x = -45 + 22$$

$$\Rightarrow x = \frac{-23}{16} = -1.44$$

$$t = -\left(\frac{-23}{16}\right) - 9 = \frac{-121}{16} = -7.56$$

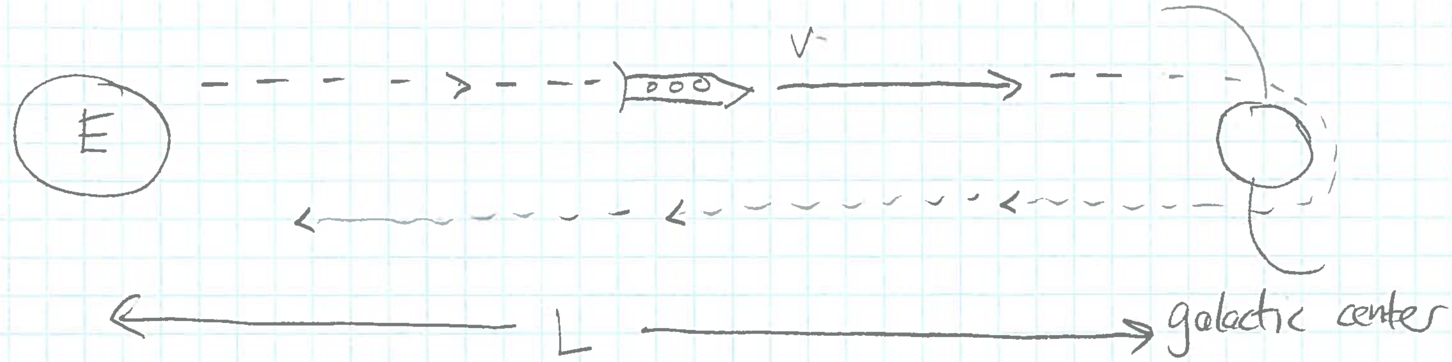
$$F = (-1.44, -7.56)$$

time acc. to Bob

g) So time of flash acc. to Biker is

$$t' = 5 + \gamma\left(t - \frac{vx}{c^2}\right) = 5 + 1.123\left(-7.56 - \frac{(0.455)(-1.4}{1^2}\right.$$

$$= \underline{\underline{-2.76 \text{ m}}}$$




Distance from Earth to center of Galaxy is

$$L = 8000\ \text{pc} = 8000\ \text{pc} * \frac{3.1\times 10^{16}\ \text{m}}{\text{pc}} = 2.48\times 10^{20}\ \text{m}$$

Corbell travels in ship at high speed on round trip. We are told the duration of the trip is

$$\Delta t_{Earth} = 3\times 10^{6}\ \text{yr}$$
$$\Delta t_{ship} = 150\ \text{yr}$$

And therefore we can estimate the ship's speed:

$$\Delta t_{Earth} = \gamma\, \Delta t_{ship}$$

$$\rightarrow \gamma = \frac{\Delta t_{Earth}}{\Delta t_{ship}} = \frac{3\times 10^{6}}{150} = 20{,}000$$

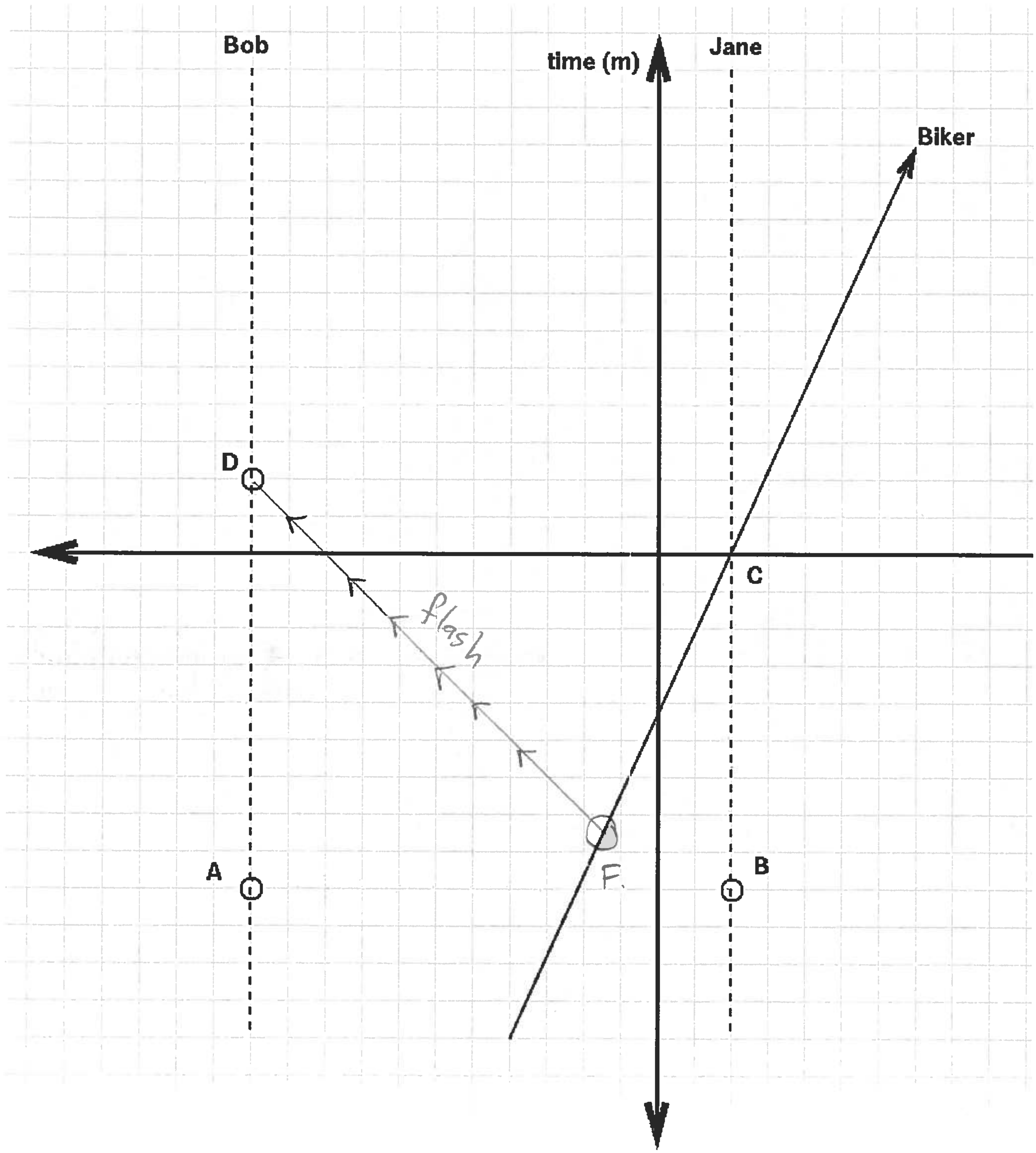
$$\rightarrow v = c\sqrt{1 - 1/\gamma^2} \simeq c\left(1 - \frac{1}{2}\frac{1}{(20{,}000)^2}\right)$$
$$= (0.999\ 999\ 998\ 75)\, c$$

At this speed, the trip should take according to people on Earth

$$\Delta t_{Earth} \approx \frac{2L}{c} \simeq \frac{5\times 10^{20}\ \text{m}}{3\times 10^{8}\ \text{s}} = 1.6\times 10^{12}\ \text{s}$$

$$\simeq 50{,}000\ \text{years}$$

Fifty thousand years is way less than three million years. So something about this story does not add up...



Bob
time (m)
Jane
Biker
D
C
flash
A
F.
B