

HW 2, #2

How far away could a supernova be, yet still appear as a neutrino source for Ice Cube?

a) Sukhbold et al., ApJ 821, 38 (2016) show total energy emitted by SNe in neutrinos as

$$E_{\nu, \text{tot}} \approx 3 \times 10^{53} \text{ erg}$$

b) If all that energy is in the form of 10 MeV neutrinos, then

$$N_{\nu} = \frac{E_{\nu, \text{tot}}}{10 \text{ MeV}}$$

$$= \frac{3 \times 10^{53} \text{ erg} \times \frac{1 \text{ eV}}{1.6 \times 10^{-12} \text{ erg}}}{10^7 \text{ eV}}$$

$$= 1.9 \times 10^{58} \text{ neutrinos}$$

c) How many electrons in Ice Cube? Assuming it is pure water ice, note that in H_2O , the mass is mostly neutrons and protons

$$2 \text{ H} = 2 \text{ p}^+$$

$$1 \text{ O} = 8 \text{ p}^+ + 8 \text{ n}$$

$$\underline{10 \text{ p}^+ + 8 \text{ n}} \quad \text{per molecule}$$

So the number of protons is $\frac{10}{18}$ of the total number of nucleons, and the mass of all the protons will be $\frac{10}{18}$ of the total mass of all the ice. So first, we compute the number of protons in all the ice.

p2

$$\begin{aligned} \text{Mass of ice} &= (1000 \text{ m})^3 \cdot (970 \text{ kg/m}^3) \\ &= 9.7 \times 10^{11} \text{ kg} \end{aligned}$$

$$\begin{aligned} \# \text{ protons} &= \left(\frac{\text{Mass of ice}}{\text{Mass of proton}} \right) \times \frac{10}{18} \\ &= 3.2 \times 10^{38} \text{ protons} \end{aligned}$$

And


$$\# \text{ electrons} = \# \text{ protons} = 3.2 \times 10^{38} \text{ electrons}$$

d) Total cross section for $e^- - \nu$ collisions is

$$\begin{aligned} \text{area } A &= (\# \text{ electrons}) (\text{cross-section}) \\ &= (3.2 \times 10^{38}) (9 \times 10^{-48} \text{ m}^2) \\ &= 2.9 \times 10^{-9} \text{ m}^2 \end{aligned}$$

e) The number of interactions with a pulse of neutrinos will be

$$\begin{aligned} N &= (\text{flux of neutrinos}) \times (\text{total cross section}) \\ &= \left(\frac{N_\nu}{4\pi d^2} \right) \cdot A \end{aligned}$$

where "d" is the distance to the explosion. 

p3

Rearrange to solve for the distance.

$$d = \sqrt{\frac{N_{\nu} \cdot A}{4\pi \cdot N}}$$

$$= \sqrt{\frac{(1.9 \times 10^{58})(2.9 \times 10^{-9} \text{ m}^2)}{4\pi \cdot 10}}$$

$$= 6.6 \times 10^{23} \text{ m} * \frac{1 \text{ pc}}{3.08 \times 10^{16} \text{ m}}$$

$$\approx 2.1 \times 10^7 \text{ pc} \approx 21 \text{ Mpc}$$

This is roughly the same as the distance to the Virgo Cluster, so maybe we could see a supernova in that cluster.